Railway Rapid Transit timetables with variable and elastic demand

David Canca[a*], Encarnación Algaba[a], Eva Barrena[b], Alejandro Zarzo[c]

[a] *E.T.S. Ingenieros, Universidad de Sevilla, Camino de los Descubrimientos s/n, 41092. Seville, Spain*
[b] *CIRRELT (Centre interuniversitaire de recherche sur les reseaux d'entreprise, la logistique et le transport), Montréal. Canada*
[c] *E.T.S.I.I. Universidad Politécnica de Madrid, José Gutiérrez Abascal, 2, 28006. Madrid, Spain*

Abstract

The aim of this paper is to provide an optimization model which, unlike the majority of previous works, considers a variable demand profile along a whole design day. So, one aspect to stand out in our model is the case of elastic demand, which leads to passengers may select an alternative transportation mode and as a result, income of service providers can be seriously compromised.

The mode choice is modeled using two alternative methods, a sigmoid function and a Logit model which influence the headway calculation. With the purpose of obtaining optimal departure times, a minimization of the loss of passengers is required. Finally, the model is applied to a real case and the computational results are shown.

Keywords: Train timetabling; elastic demand, mixed-integer nonlinear programming

1. Introduction

Timetabling has been widely analysed in scientific literature. In fact, it may be a determinant aspect on users' quality of service perception. Cacchiani, (2008), presented a survey about optimization models, focussing on the role of timetabling inside the classic hierarchical planning approach. However, when designing railway

* Corresponding author. Tel.: +3-495-448-6197; fax: +3-495-448-7329.
E-mail address: dco@us.es

timetables, most of previous approaches are centred on the case of uniform demand. In this case, according to the random incidence theorem (Larson and Odoni, (1981)), regular schedules are the best solution if waiting time of passengers is minimized. Interesting exceptions to consider are the work of Chierici et al., (2004), and the posterior extension published by Cordone and Redaelli, (2011), where the common assumption of regular demand is relaxed and the influence between timetable quality and captured demand is considered. In this context, once the timetable is obtained, some trains are removed to accomplish the transport demand for each non-peak hour.

In this paper, our main aim is to provide railway timetables applying an optimization model which deals with a variable demand profile during a long planning horizon (e.g. a complete design day) and considering the case of elastic demand. Therefore, passengers can select an alternative transportation mode and, as a consequence, income of service providers can be seriously compromised. Mode choice is modelled by two alternative methods, a sigmoid function and a Logit model that influences headway calculation. Thus, a minimization of the loss of passengers is used to obtain optimal departure times. Computational results for a real case study are provided, namely, the model is applied to a piece of the C5 line belonging to the Madrid suburban railway network. The experiments are implemented for the two different mode rejection probability approaches under consideration (sigmoid/Logit model) and some comments about the characteristics and usage of both methods is performed.

2. Timetabling optimization model with variable demand

In this section, we present an enhanced optimization model based on a previous work (Canca et al., (2011)) in order to determine the train departure times from the origin station as well as the arrival/departure times at/from each one of the stations, for each train, along a line. As mentioned before, this approach, unlike other models in the existent literature, is based on the cumulative demand approximation. In fact, a generic continuous representation of the mobility demand $OD(t)$ along the daily planning horizon is considered. Namely, $OD(t)$ is a square matrix as many rows and columns as stations in the network. Each entry, denoted by $f_{ij}(t)$ represents the continuous daily evolution of the travel demand from station i to station j.

Although these demand curves can be quite different from a problem to another, a common and relevant characteristic of all of them is the existence of certain demand peaks (local maxima) in certain instants of time, along the day. These peaks are associated to rush hours and generally reduced to two or three per day. Following the ideas already described in Canca et al., (2011) in our approach to the passengers demand, we consider the cumulative or aggregated demand $F_{ij}(t)$, given by,

$$F_{ij}(t) = \int_{o}^{t} f_{ij}(s)\, ds. \tag{1}$$

As the demand will be used to obtain train departure times and provide measures to characterize the performance of timetables, an analytical expression of it would be of interest. Taking into account the shape of the cumulative demand between each pair of stations, it makes sense to consider an approximation given by a linear combination of a variable number, M, of sigmoid curves, i.e.,

$$F_{ij}(t) = \sum_{r=1}^{M} \frac{K_{ij}^{\ r}}{1+e^{-\beta_{ij}^{\ r}(t-x_{ij}^{\ r})}}. \tag{2}$$

This approximation is fully characterized by a number of parameters (including M) which are determined by solving a set of appropriate least squares minimization problems (see Canca e al., (2011)). Once the cumulative demand curve is obtained, it is used as data to model passenger behavior and determine the optimal timetable. We emphasize that this model can be applied to other kind of characterizations of the demand, even it can be adaptable for its application to problems in which discrete demand functions are used, as the ones typically obtained from mobility surveys.

In order to describe the optimization model, in the following subsections, the notation, constraints and a discussion on the objective will be introduced.

2.1. Notation

Next, we comment the more relevant aspects of the model, and the enhancements to work with elastic demand. A more detailed analysis corresponding to the inelastic case can be found in Canca et al., (2011). In the following table, the notation of sets, parameters and variables, used in the model, is given for a generic line.

Table 1. Symbols and definitions

	Sets and parameters
$H := \{1,2,...,S\}$	Set of stations.
$N := \{1,2,...,K\}$	Set of trains.
$L := \{1,2,...,S-1\}$	Set of segments between each pair of stations.
T	Planning horizon, usually one day (time units).
$lng(i,i+1)$	Length of the segment corresponding to stations i and i+1.
$Mint_{stop}$, $Maxt_{stop}$	Lower and upper bounds for stop times at stations.
$t_{saf}(i,k)$	Safe headway time after departure of train k from station i.
$t_{acc}(i,i+1)$	Acceleration time needed to reach pure running speed after station i.
$t_{dec}(i,i+1)$	Deceleration time needed to stop from running speed before station i+1.
g	Passenger flow rate, pax/min. Door open and close times are considered negligible.
	Variables
t_i^k	Departure time of train k at the i-th station.
$\overline{V_k}(i,i+1)$	Unit travel time of train k in the segment between stations i and i+1.
FS_i^k	Available capacity in the k-th train when it leaves the i-th station.
CAP_k	Capacity of train k.
$t_{stop}(i,k)$	Stop time of train k at station i.
β_i^k	Binary variable which takes value 1 if train k stops at station i.
	Arrivals
$N_{ij}^k = F_{ij}^{[t_i^{k-1},t_i^k]}$	Number of people who arrive at the i-th station with destination to the j-th station during the time interval $[t_i^{k-1},t_i^k]$.
NAD_i^k	Number of people who arrive at the i-th station during the time interval $[t_i^{k-1},t_i^k]$.
NAO_j^k	Number of people who arrive at any station i ($i \leq j$) with destination to the j-th station during the time interval $[t_i^{k-1},t_i^k]$.
ns_{ij}^k	Number of people who arrive at the i-th station with destination to the j-th station during the time interval $[t_i^{k-1},t_i^k]$ and get on the k-th train.
ne_{ij}^k	Number of people who arrive at the i-th station with destination to the j-th station during the time interval $[t_i^{k-1},t_i^k]$ and do not get on the k-th train.
	Waiting
E_{ij}^k	Number of people who arrive at the i-th station with destination to the j-th station before t_i^{k-1} (i.e., during the interval $[0,t_i^{k-1}]$) and are waiting on platform before the departure of the

	k-th train.
EAD_i^k	Number of people who arrive at the i-th station before t_i^{k-1} (i.e., during the interval $[0,t_i^{k-1}]$) and are waiting on platform before the departure of the k-th train.
EAO_j^k	Number of people who arrive at any station i ($i \le j$) before t_i^{k-1} (i.e., during the interval $[0,t_i^{k-1}]$) and are waiting on platform before the departure of the k-th train with destination to the j-th station.
ee_{ij}^k	Number of people who arrive at the i-th station with destination to the j-th one, before t_i^{k-1} (i.e., during the interval $[0,t_i^{k-1}]$) and do not get on the k-th train.
es_{ij}^k	Number of people who arrive at the i-th station with destination to the j-th one, before t_i^{k-1} (i.e., during the interval $[0,t_i^{k-1}]$) and get on the k-th train.
Boarding	
S_{ij}^k	Number of people who get on the k-th train at the i-th station with destination to the j-th one.
SAD_i^k	Number of people who get on the k-th train at the i-th station.
SAO_j^k	Number of people who get on the k-th train with destination to the j-th station.

We assume known the length and the minimum and maximum speed limitations in every segment, $l \in L$..

2.2. Constraints

The relationship between departure times at consecutive stations along the line is shown in Equation (3). An interesting extension, as proposed by Zhou and Zhong, (2005), considering acceleration and deceleration times, can be easily incorporated to this model by adding the term $(t_{acc} + t_{dec})\beta_{i+1}^k$ to the left side of constraints given in (4). In this case, $\overline{V_k}(i,i+1)$ refers to the "pure running" speed. Equation (5) defines the minimum headway between consecutive trains k and k+1.

$$t_{i+1}^k = t_i^k + t_{stop}(i+1,k) + lng(i,i+1)\overline{V_k}(i,i+1), \quad \forall k \in N,\ i \in H \setminus \{S\}. \tag{3}$$

$$t_i^{k+1} - t_{stop}(i,k+1) \ge t_i^k + t_{saf}(i,k), \quad \forall k \in N \setminus \{K\},\ i \in H. \tag{4}$$

$$Mint_{stop}\ \beta_i^k \le t_{stop}(i,k) \le Maxt_{stop}\ \beta_i^k, \quad \forall k \in N,\ i \in H. \tag{5}$$

Equation (6) implements the relationship between the departure times and passenger demand. Equation (7) represents the sum of arrivals for passengers coming to station i with destination to station j (j>i). Equation (8) computes the sum of passenger arrivals from several stations (i<j) to a certain one, denoted by index j.

$$N_{ij}^k = F_{ij}^{[t_i^{k-1},t_i^k]} = \sum_{r=1}^{M} \frac{K_{ij}^r}{1+e^{-\beta_{ij}^r(t_i^k - x_{ij}^r)}} - \sum_{r=1}^{M} \frac{K_{ij}^r}{1+e^{-\beta_{ij}^r(t_i^{k-1} - x_{ij}^r)}}, \quad \forall i,j \in H,\ i<j,\ k \in N. \tag{6}$$

$$NAD_i^k = \sum_{\{j \in H\ :\ j>i\}} N_{ij}^k, \quad \forall i \in H \setminus \{S\},\ k \in N. \tag{7}$$

$$NAO_j^k = \sum_{\{i \in H\ :\ i<j\}} N_{ij}^k, \quad \forall j \in H \setminus \{1\},\ k \in N. \tag{8}$$

With the objective of calculating non-served passenger demand, the number of people who have arrived is decomposed into the sum of those passengers who will manage to get on the k-th train $\left(ns_{ij}^k\right)$ and those who will not achieve to do it $\left(ne_{ij}^k\right)$.

$$N_{ij}^k = ns_{ij}^k + ne_{ij}^k, \quad \forall i, j \in H,\ i < j,\ k \in N. \tag{9}$$

In the same way, the number of passengers who wait for train *k*, having arrived at the platform before t_i^{k-1}, is expressed as the sum of those who manage to get on train *k* and those who do not achieve it, ee_{ij}^k, Equation (10). Waiting variables are also aggregated by destinations and origins, as we did above with arrivals, obtaining variables like EAD_i^k (Equation (11)) and EAO_j^k (Equation (12)) to balance waiting people between two consecutive trains.

$$E_{ij}^k = es_{ij}^k + ee_{ij}^k, \quad \forall i, j \in H,\ i < j,\ k \in N. \tag{10}$$

$$EAD_i^k = \sum_{\{j \in H\ :\ j > i\}} E_{ij}^k, \quad \forall i \in H \setminus \{S\},\ k \in N. \tag{11}$$

$$EAO_j^k = \sum_{\{i \in H\ :\ i < j\}} E_{ij}^k, \quad \forall j \in H \setminus \{1\},\ k \in N. \tag{12}$$

Constraints (13) and (14) are used to balance people by origin and destination.

$$EAO_i^{k+1} = EAO_i^k + NAO_i^k - SAO_i^k, \quad \forall i \in H \setminus \{1\},\ k \in N \setminus \{K\}. \tag{13}$$

$$EAD_i^{k+1} = EAD_i^k + NAD_i^k - SAD_i^k, \quad \forall i \in H \setminus \{S\},\ k \in N \setminus \{K\}. \tag{14}$$

Equation (15) describes the number of passengers who get on the *k*-th train. Aggregating people by destination, SAD_i^k and origin SAO_j^k, Equations (16) and (17) are obtained, respectively.

$$S_{ij}^k = ns_{ij}^k + es_{ij}^k, \quad \forall i, j \in H,\ i < j,\ k \in N. \tag{15}$$

$$SAD_i^k = \sum_{\{j \in H\ :\ j > i\}} S_{ij}^k, \quad \forall i \in H \setminus \{S\},\ k \in N. \tag{16}$$

$$SAO_j^k = \sum_{\{i \in H\ :\ i < j\}} S_{ij}^k, \quad \forall i \in H \setminus \{1\},\ k \in N. \tag{17}$$

Now, it is possible to balance the capacity of each train at each one of the stations, using the variables defined above and the available capacity variable, FS_i^k. The balance of the capacity of each train at each station is shown in Equations (18) and (19).

$$FS_i^k = FS_{i-1}^k + SAO_i^k - SAD_i^k, \quad \forall i \in H \setminus \{1\},\ k \in N. \tag{18}$$

$$FS_1^k = CAP_k - SAD_1^k, \quad k \in N. \tag{19}$$

At this point, it may be determined appropriately stop intervals considering the minimum between the time needed to occupy the train available capacity (Equations (20), (21)) or the passenger flow time according to people getting on each train, Equation (22).

$$t_{stop}(i,k) \le \mathrm{Mint}_{stop} + g\ FS_{i-1}^k, \quad \forall k \in N,\ i \in S \setminus \{1\}. \tag{20}$$

$$t_{stop}(1,k) \le \mathrm{Mint}_{stop} + g\ CAP_k, \quad \forall k \in N. \tag{21}$$

$$t_{stop}(i,k) \le \mathrm{Mint}_{stop} + g\ SAD_i^k, \quad \forall k \in N,\ i \in S \setminus \{1\}. \tag{22}$$

$$N_{ij}^k, S_{ij}^k, E_{ij}^k, ns_{ij}^k,\ ne_{ij}^k, es_{ij}^k, ee_{ij}^k, EAD_i^k, EAO_i^k, FS_i^k, t_i^k, NAD_i^k, NAO_i^k, SAD_i^k, SAO_i^k \ge 0, \quad \forall i, j \in H,\ i \ne j,\ k \in N.$$

$$E_{ij}^1 = 0, \quad \forall i, j \in H,\ i \ne j, \quad CAP_k \ge 0,\ \forall k \in N,\ \beta_i^k(0,1), \quad \forall i \in H, \quad k \in N.$$

2.3. Waiting time and mode rejection

With the aim of obtaining accurate timetables, we will consider a main objective-function that minimizes the total loss of passengers (*LOP*) during the whole planning horizon. Since in the case of variable demand it is not possible to formulate an explicit relationship between average waiting time and non-regular departure times, as we pointed out above, it is necessary to express directly the average waiting time considering arrivals between each pair of trains and departure times at each station. So, assuming an inelastic demand, the objective function would be the minimization of the average waiting time, as it is shown in Equation (23).

$$Min \frac{1}{\sum_{i=1}^{S-1}\sum_{j=i+1}^{S} F_{ij}^{[0,T]}}\left[\sum_{i=1}^{S-1}\sum_{k=1}^{K}(t_i^k - t_i^{k-1})EAD_i^k + \frac{1}{2}\sum_{i=1}^{S-1}\sum_{k=1}^{K}(t_i^k - t_i^{k-1})NAD_i^k\right]. \tag{23}$$

As we mentioned above, Equation (6) defines the relationship between passenger arrival and departure time of two consecutive trains. When we consider elastic demand, long interdeparture times can affect passenger demand. Therefore, a fraction of passengers waiting for the next train could decide to abandon the station and take an alternative transport mode. Figure 1 shows this situation and illustrates how *LOP* is now taken into account in the balance constraints (Equations (13),(14)).

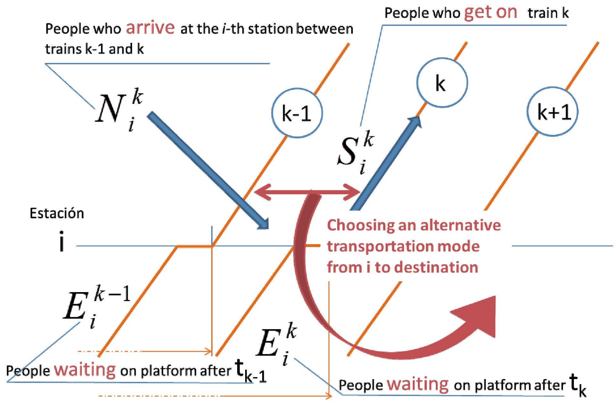


Figure 1. Relationship among arrival, boarding and waiting variables

Following the demand model described previously, *LOP* is illustrated in Figure 2. The shaded area represents the number of passengers that reject waiting for the next train and choose a different alternative. In our approach, a first method to compute the probability of passenger leaving the rail mode at station *i* (with destination to station *j*) after arrival of train *k* is defined as:

$$Prob_{ij}^k = \left(\frac{1}{1-e^{\lambda}}\right)\left(1-e^{\lambda x_{ij}^k}\right). \tag{24}$$

Where x_{ij}^k represents the fraction of the interdeparture time between train *k*-1 and train *k* and the expected time to reach the destination (waiting time plus with travel time) as described in Equation (25). On the other hand, the parameter λ reflects the degree of passenger impatience. Note that the bigger the degree of the passenger impatience is, the higher the probability of changing to an alternative mode as it is shown in Figure 3.

$$x_{ij}^k = \frac{\frac{1}{2}(t_i^k - t_i^{k-1})}{\frac{1}{2}(t_i^k - t_i^{k-1}) + \sum_{i}^{j-1}\frac{Lenght(i)}{v(i,i+1)}}. \tag{25}$$

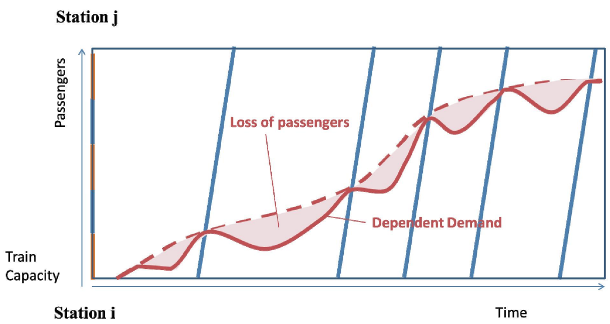


Figure 2. Loss of passengers

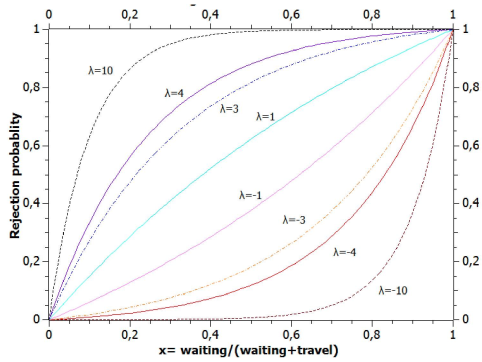


Figure 3. Rejection probability function

In addition to this, note that the rejection probability diminishes when the length of the trip is shorter. Next, we propose a second objective function to obtain the optimal timetable that minimizes the loss of passengers along the whole line:

$$Min\sum_{i=1}^{S-1}\sum_{k=1}^{K} LOP_i^k = Min\sum_{i=1}^{S-1}\sum_{j=i+1}^{S}\sum_{k=1}^{K}\mathrm{Prob}_{ij}^k NAD_i^k. \qquad (26)$$

A second possibility, considering explicitly the existence of an alternative mode, consists of using a Logit model. Let u^r, u^a be the generalized cost of the railway mode and the corresponding cost of the alternative one, respectively. Then, the probability of choosing the railway mode to complete the journey when a user waits for train *k* to go from station *i* to station *j* can be expressed as:

$$p(u_{ij}^{kr},u_{ij}^{a}) = \frac{e^{-(\alpha_{ij}^{kr}+\beta u_{ij}^{kr})}}{e^{-(\alpha_{ij}^{a}+\beta u_{ij}^{a})}+e^{-(\alpha_{ij}^{kr}+\beta u_{ij}^{kr})}} = \frac{1}{1+e^{-\left[\left(\alpha_{ij}^{a}-\alpha_{ij}^{kr}\right)+\beta\left(\alpha_{ij}^{r}-u_{ij}^{kr}\right)\right]}} = \frac{1}{1+e^{-\left[\alpha_{ij}^{k}+\beta\left(u_{ij}^{a}-u_{ij}^{kr}\right)\right]}}. \qquad (27)$$

The generalized cost of trip from *i* to *j* in using the railway line, could be obtained as a function of the inter-departure time between trains *k* and *k+1* and the travel time:

$$u_{ij}^{kr} = \frac{1}{2}(t_i^k - t_i^{k-1}) + \sum_{i}^{j-1}\frac{Length(i)}{v(i,i+1)}.$$

From a practical point of view, we will consider only two transport modes competing along the line, railway and bus. Moreover, we assume that this competition affects equally each pair (*i,j*) of stations and that in case of similar generalized cost, there are no a special tendency to one specific mode. In these circumstances, expression (27) can be simplified, obtaining:

$$p(u_{ij}^{kr},u_{ij}^{a}) = \frac{1}{1+e^{-\beta\left(u_{ij}^{a}-u_{ij}^{kr}\right)}}. \qquad (28)$$

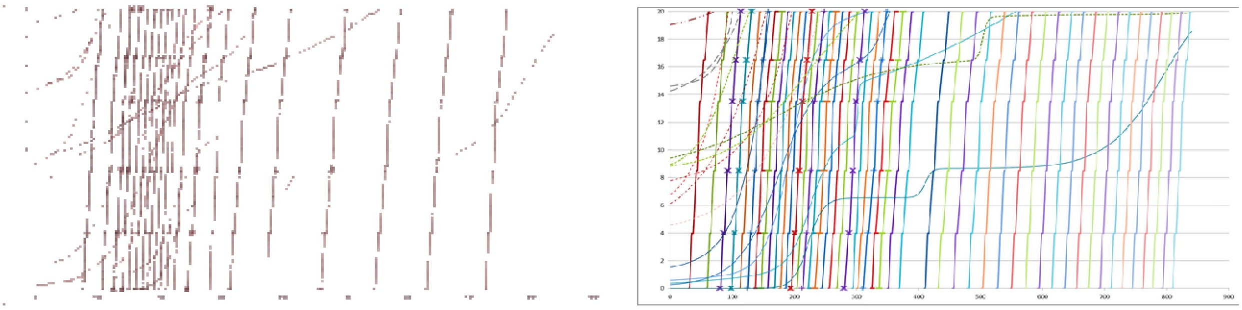
Therefore,

$$Prob_{ij}^{k} = 1 - p(u_{ij}^{kr},u_{ij}^{a}). \qquad (29)$$

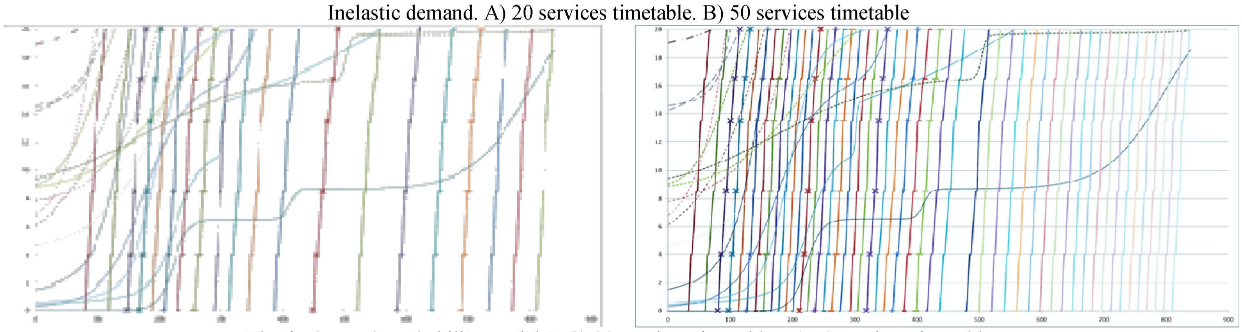
The main difference between (24) and (29) is that in the second expression, we need to compare railway trip time (waiting plus travel time) with the trip time of the alternative mode, whereas the first probability model only considers passenger impatience. Another interesting point concerning these methods is that the second model gives rise to a non-linear smooth non-convex objective function whereas the first probability approach, depending on the value of the parameter λ, defines a smooth non-linear convex or non-convex objective function.

3. Elastic demand model. Computational results

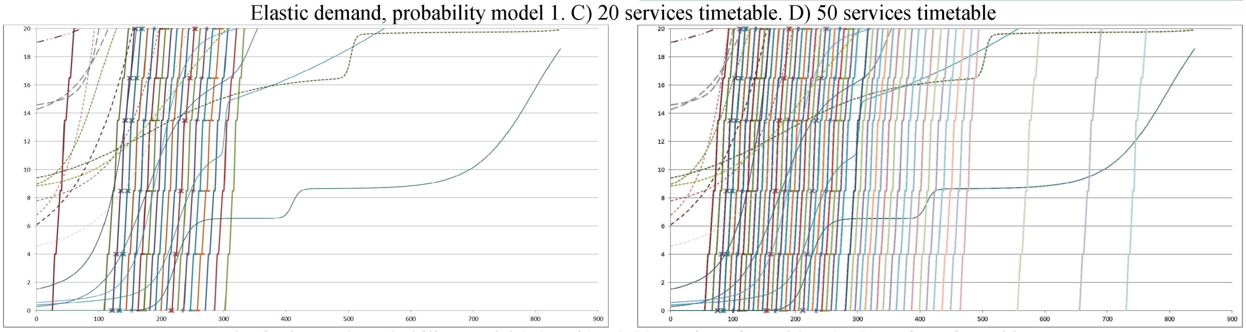
In order to illustrate the introduced model, an application on a piece of the C5 line belonging to the Madrid suburban railway network has been used. With this aim, we first used surveys data (offered by RENFE) to adjust the cumulative demand functions among six stations of the line as in Canca et al., (2011). Next, for both cases, inelastic and elastic demand, we have computed different timetables varying the number of allowed services. We assume that the trains have a fixed capacity, equals to 400 passengers and we consider a global demand of 11581 passengers for an eight hours planning horizon. Figure 4 shows two examples corresponding to 20 and 50 services, respectively, where the total loss of passengers (*LOP*) has been obtained using the first probability model with $\lambda = -3$. The model has been implemented using *GAMS* and solved by a branch and bound procedure based on the branch and cut module *Cbc* and the cut-generation library *Cgl* included in the *COIN-OR* distribution. To solve the continuous *NLPs,* the interior-point solver *Ipop,t* by Watcher and Biegler, (2005), has been used. A detailed discussion on *COIN-OR* open source library can be found in *www.coin-or.org*.



Inelastic demand. A) 20 services timetable. B) 50 services timetable



Elastic demand, probability model 1. C) 20 services timetable. D) 50 services timetable



Elastic demand, probability model 2 (Logit). E) 20 services timetable. F) 50 services timetable

Figure 4. Timetables with and without considering elastic demand at stations

Notice that in all the experiments carried out, the elastic demand inclusion gives rise to a more regular timetable, even in the case of variable demand, where regular timetables are sub-optimal from the average waiting time criteria point of view. Moreover, using the first probability model (Figure 4.C-D), this tendency is greater when the degree of impatience (parameter λ) is higher. Clearly, longer interdeparture times force passengers to choose an alternative mode of transport (not implicitly considered by the first model) for certain trips. In fact, interdeparture times decrease in periods where cumulative demand increases. These results suggest the convenience of using near-regular timetables in different intervals along the whole planning horizon when rejecting mode probabilities are considered. This first probability model reflects a situation with no clear competitive modes along the line (or with a high number of alternative possibilities corresponding to certain pairs in which case, obtaining data to be included in the design procedure, results difficult).

The second probability model is applied in case of existence of a secondary alternative transportation mode. First, note that in order to perform the experiments it is necessary to know implicitly the travel time of the complementary mode for each origin destination pair. In this paper, a complete regular timetable has been considered for the bus mode to compute waiting times. Moreover, for each origin-destination pair, the pure running time of bus has been randomly generated with values which vary from 30% to100% greater than the railway pure running times (this is a reasonable assumption, because bus competes with private vehicle on surface whereas railway does not share infrastructure with any other mode, in fact, commercial speeds of urban/interurban buses are usually less than half of the rapid transit ones). Experiments are carried out with the same number of services in both modes.

It is interesting to highlight that results of the Logit model are quite different: In the case of only a few services (i.e., 20 services, Figure 4E), the model allocates all the services in the time interval corresponding to the main peak zone of the demand shape. Note that in this approach, railway is competing with bus and shorting drastically the headway in the peak demand interval makes easier the capture of a high percentage of passengers with respect to the alternative mode, even if it was not necessary to serve long periods outside of the peak demand area.

However, this behavior is smoothed in case of 50 services (Figure 4F). In this situation, three different zones can be observed. The first one corresponds to the peak demand interval and there, the model reduces as much as possible the headway to capture a high percentage of the demand. The second one corresponds to a higher demand interval with less slope, the model proposes a greater headway, this is logical because the cumulative demand is lower than in the peak zone (and due to safety constraints it is not possible to capture more people in the peak zone). Finally, some trains are allocated in the rest of the planning horizon with long headways and probably with the hope to capture only a few passengers. Clearly, the results of the Logit model can be interpreted as the need of a high number of services to accommodate the daily demand. Of course, this conclusion must be also considered under an economic criteria, in fact, at equilibrium each mode will capture a fraction of the demand and this fraction can be imposed as a global constraint, obtaining, in this case, timetables that guarantee certain level of share.

4. Conclusions

This paper describes the determination of railway rapid transit systems timetables relaxing the unrealistic assumption of uniform demand behaviour and allowing for the analysis of a full day operation. A non-linear integer programming model is used in order to schedule railway services with the objective of attending a given variable demand under the minimum average waiting time criteria. The model is enhanced considering the case of elastic demand. Two different mode rejection probability models have been considered, the first one using a sigmoid function and the second one a Logit model. The application of the model to the case of elastic demand, leads to the possibility that passengers may choose an alternative transportation mode and as a result, income of service providers can be seriously compromised. Several comments, supported on a set of numerical experiments, about the advantages/disadvantages and applicability of each one are also accomplished. Results of these models should be accompanied of a detailed economic analysis and, in this case, the timetable model can be applied considering global conditions about the market share, designing timetables with the aim of conserving or even improving the percentage of the captured demand.

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